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# Modified Robertson–Walker Metric and Matter–Antimatter Asymmetry

**L. Corsiglia**

*4043 North Paulina Street, Chicago, Illinois 60613*

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A modified Robertson–Walker metric is used to obtain solutions for matter density in the time interval $-\infty < t < +\infty$. The manner in which a complex Coulomb potential might produce the observed matter–antimatter asymmetry of the universe is described.

## 1. THE METRIC

The metric chosen for study is

$$ds^2 = e^{f(x^0)}(dx^0)^2 - e^{h(x^0)} \frac{dr^2 + r^2\, d\theta^2 + r^2 \sin^2(\theta)\, d\phi^2}{\left[1 + (k/4)(r^2/L^2)\right]^2} \tag{1}$$

With $f(x^0) = 0$, equation (1) becomes the usual Robertson–Walker metric. The field equations chosen for study are those with zero cosmological term

$$R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi G}{c^2} T_{ij} \tag{2}$$

The energy–momentum tensor is

$$T^i_j = \begin{pmatrix} D(t) & 0 & 0 & 0 \\ 0 & -p/c^2 & 0 & 0 \\ 0 & 0 & -p/c^2 & 0 \\ 0 & 0 & 0 & -p/c^2 \end{pmatrix} \tag{3}$$

in which $D=D(t)$ represents average matter density and $p=p(t)$ represents average internal pressure of matter (or energy). The notation for derivatives is $f'=df/dx^0=df/c\,dt=\dot{f}/c$. (Adler et al., 1975)

The result from equations (1), (2), and (3) is

$$h'=-\frac{2D'}{3(D+p/c^2)} \tag{4}$$

$$e^f=\frac{4D'^2}{9(D+p/c^2)^2}\left(\frac{32\pi GD}{3c^2}-\frac{4ke^{-h}}{L^2}\right)^{-1} \tag{5}$$

Solutions for density $D$ and temperature $T$, using equations (4) and (5), will be obtained for various eras in the time interval $-\infty<t<+\infty$. Hence, the possibility of mathematical and physical solutions prior to the big bang $(t=0)$ of standard model cosmology will be investigated. In addition, a method of joining the solutions for density immediately before and after $t=0$ will be presented for the $k=0$ case in the elementary particle model (Weinberg, 1972).

Table I presents a description of the seven eras studied. $N$ represents, in the elementary particle model, $\mathcal{N}/2$ different kinds of elementary particles, counting spin states and antiparticles separately, and counting fermions as 7/8 of a particle (Weinberg, 1972). $N=9/2$ for $10^{10}$ K$\simeq T<10^{12}$ K (Weinberg, 1972). For eras 1 and 2, the usual Robertson–Walker metric is used so that $e^f=1$ for them. From equation (5), one can determine that $e^f=2/9$ and $1/N$ for eras 3 and 4, with $k=0$, in comparison to a pure photon era. $B$ and $A$ in the $e^h$ column are integration constants. The coordinate velocity of the photon will be discussed later.

## 2. ERA 1

$B_1=1/24\pi G$, $B_2=3c^2/8\pi GL^2B^{2/3}$, and $e^h=R^2(t)/L^2$ are used.

**(i) $k=0$:**

$$D=\frac{1}{6\pi Gt^2} \tag{6}$$

$$R=L(6\pi GB)^{1/3}t^{2/3} \tag{7}$$

TABLE I. Description of Eras Studied

| Era | $t$ | Equation of state | Matter density, $D$ | $k$ | $e^f$ | $e^h$ | Coordinate speed of photon, $d\sigma/dt$ |
|---|---|---|---|---|---|---|---|
| 1. Pressure free | $>0$ | $p=0$ | — | $0, \pm 1$ | 1 | $(B/D)^{2/3}$ | — |
| 2. Radiation dominated | $>0$ | $p/c^2 = D/3$ | $aT^4/c^2$ | $0, \pm 1$ | 1 | $(A/D)^{1/2}$ | — |
| 3. $10^{10} \simeq T < 10^{12}$ K | $>0$ | $p/c^2 \simeq D/3$ | $9aT^4/2c^2$ | 0 | $2/9$ | $(A/D)^{1/2}$ | — |
| 4. $T > 10^{12}$ K | $>0$ | $p/c^2 \simeq D/3$ | $NaT^4/c^2$ | 0 | $1/N$ | $(A/D)^{1/2}$ | — |
| 5. $0 < T < 10^{10}$ K | $<0$ | $p/c^2 = D/3$ | $aT^4/c^2$ | $0, \pm 1$ | — | $(A/D)^{1/2}$ | $c$ |
| 6. $10^{10} \simeq T < 10^{12}$ K | $<0$ | $p/c^2 \simeq D/3$ | $9aT^4/2c^2$ | 0 | — | $(A/D)^{1/2}$ | $(2/9)^{1/4}c$ |
| 7. $T > 10^{12}$ K | $<0$ | $p/c^2 \simeq D/3$ | $NaT^4/c^2$ | 0 | — | $(A/D)^{1/2}$ | $(1/N)^{1/4}c$ |

**(ii) $k=-1$:**

$$\frac{\left(B_2^{-1}D^{1/3}+1\right)^{1/2}}{2B_2^{-1}D^{1/3}}-\frac{1}{2}\log_e\frac{1+\left(B_2^{-1}D^{1/3}+1\right)^{1/2}}{B_2^{-1/2}D^{1/6}}=\left(\frac{B_2^3}{36B_1}\right)^{1/2}t \tag{8}$$

$$R(\tau)=\frac{B^{1/3}L}{2B_2}(\cosh 2\tau-1) \tag{9}$$

$$ct(\tau)=\frac{B^{1/3}L}{2B_2}(\sinh 2\tau-2\tau) \tag{10}$$

**(iii) $k=+1$:**

$$-\frac{\left(B_2^{-1}D^{1/3}-1\right)^{1/2}}{2B_2^{-1}D^{1/3}}+\frac{1}{2}\sin^{-1}\frac{1}{B_2^{-1}D^{1/6}}=\left(\frac{B_2^3}{36B_1}\right)^{1/2}t \tag{11}$$

$$R(\tau)=\frac{B^{1/3}L}{2B_2}(1-\cos 2\tau) \tag{12}$$

$$ct(\tau)=\frac{B^{1/3}L}{2B_2}(2\tau-\sin 2\tau) \tag{13}$$

Comparing equations (7), (9), and (12) with the standard derivations (Adler et al.), one finds

**(i) $k=0$:**

$$LB^{1/3}=\left(\frac{3c^2D_0}{8\pi G}\right)^{1/3} \tag{14}$$

**(ii) $k=-1$:**

$$LB^{1/3}=\left(\frac{3c^2}{8\pi G}\right)^{1/3}\left[\frac{2q_0c}{(1-2q_0)^{3/2}H}\right]^{1/3} \tag{15}$$

**(iii) $k=+1$:**

$$LB^{1/3}=\left(\frac{3c^2}{8\pi G}\right)^{1/3}\left[\frac{2q_0c}{(2q_0-1)^{3/2}H}\right]^{1/3} \tag{16}$$

in which $D_0$ is an undetermined constant, $q_0$ is the deceleration parameter, and $H$ is Hubble's constant.

From equation (11), the minimum density in a $k=+1$ universe is given by $B_2^3$:

$$D_{\min}=1.4\times10^{-30}\frac{(2q_0-1)^3}{q_0^2}\ \mathrm{g/cm^3} \tag{17}$$

For $q_0=1$, $D_{\min}=1.4\times10^{-30}$ g/cm³, reached at $t=5.6\times10^{10}$ years.

## 3. ERA 2

Letting $A_1=3/128\pi G$ and $A_2=3c^2/8\pi GL^2A^{1/2}$,

$$T=\left(\frac{4A_1c^2}{a}\right)^{1/4}t^{-1/2}\left(1-k\frac{A_2t}{8A_1^{1/2}}\right)^{-1/2} \tag{18}$$

$$D=\frac{3}{32\pi Gt^2}\left(1-k\frac{A_2t}{8A_1^{1/2}}\right)^{-2} \tag{19}$$

## 4. ERA 3

Only $k=0$ is used, and only the density equation written out, for eras 3, 4, 6, and 7:

$$D=\frac{9}{2}\left(\frac{3}{32\pi Gt^2}\right) \tag{20}$$

## 5. ERA 4

We have

$$D=\frac{3N}{32\pi G(t+h_1)^2},\qquad h_1>0 \tag{21}$$

in which $h_1$, the constant of integration, is not set equal to zero in order to join the solution of era 4 to the solution of era 7 at $t=0$. Eras 4 and 7 are both described by the same $N$, making physical continuity across $t=0$ plausible. (Sections 11 and 12 discuss the possibility of $N$ changing near $t=0$.)

## 6. ERA 5

Equations (1), (4), and (5) yield, for the coordinate speed squared of a single photon,

$$\frac{1}{c^2}\left(\frac{d\sigma}{dt}\right)^2=\frac{A_1}{A^{1/2}}\frac{\dot{D}^2}{D^{5/2}}\left(1-\frac{A_2 k}{D^{1/2}}\right)^{-1} \tag{22}$$

It is assumed that $d\sigma/dt=c$ in era 5. From equation (22), one can determine that $d\sigma/c\,dt=(2/9)^{1/4}$ and $(1/N)^{1/4}$ for eras 6 and 7, with $k=0$, in comparison to a pure photon era.

**(i) $k=0$:**

$$T=-\left(\frac{16A_1 c}{A^{1/2}a^{1/2}}\right)^{1/2}\frac{1}{t} \tag{23}$$

$$D=\left(\frac{256A_1^2}{A}\right)\frac{1}{t^4} \tag{24}$$

$T$ and $D$ each begin at zero ($t=-\infty$) and each approach a singularity at $t=0$. But pair production interferes at $T=6\times10^9$ K as era 6 is approached.

**(ii) $k=-1$:**

$$T=-\left(\frac{A_2 c}{a^{1/2}}\right)^{1/2}\operatorname{csch}\left[\left(\frac{A^{1/2}A_2}{16A_1}\right)^{1/2}t\right] \tag{25}$$

$$D=A_2^2\left\{\operatorname{csch}\left[\left(\frac{A^{1/2}A_2}{16A_1}\right)^{1/2}t\right]\right\}^4 \tag{26}$$

$T$ and $D$ vary as in the $k=0$ case.

**(iii) $k=+1$:**

$$T=-\left(\frac{A_2 c}{a^{1/2}}\right)^{1/2}\csc\left[\left(\frac{A^{1/2}A_2}{16A_1}\right)^{1/2}t\right] \tag{27}$$

$$D=A_2^2\left\{\csc\left[\left(\frac{A^{1/2}A_2}{16A_1}\right)^{1/2}t\right]\right\}^4 \tag{28}$$

Since equation (27) allows oscillating and negative $T$ and since equations (12) and (13) are cyclic in era 1 for $k=+1$, one likely need not consider the possibility of a solution for $t<0$ for $k=+1$.

## 7. ERA 6

We have

$$D=\frac{9}{2}\left(\frac{256A_1^2}{A}\right)\frac{1}{t^4} \tag{29}$$

## 8. ERA 7

We have

$$D=\frac{256A_1^2N}{A(t+h_2)^4}, \qquad h_2<0 \tag{30}$$

in which $h_2$ is the nonzero constant of integration for era 7.

## 9. $D(t=0)$

Setting the density functions and their second derivatives for eras 4 and 7 equal to each other at $t=0$, one obtains

$$D(t=0)=0.7NA \text{ g/cm}^3 \tag{31}$$

in which the integration constant $A$ represents a density. (See Table I.) If $A$ represents the density at the end of a given era (for example, the density at which electron–positron production occurs at the end of era 5), then $A$ could represent the density at which free quarks are produced from baryons at the end of era 7 in equation (31). Since first derivatives cannot be matched at $t=0$, a discontinuity in slope still occurs at $t=0$.

## 10. COMPLEX COULOMB POTENTIAL

The use of complex space-time coordinates has been introduced into physics (Thiess, 1968; Newman, 1973; Adler et al., 1975; Corsiglia, 1975 and 1976). The complex Coulomb potential is given by $e/x$ with $x=r+ib$, $r$ the usual radial coordinate and $b$ real. The complex electric field is given by $E_x = e/x^2$ and the energy density by $w=|E_x|^2/8\pi$. Integrating over all space to find the self-energy, one obtains

$$W=\frac{\pi e^2}{8b} \tag{32}$$

If a classical point particle is described by $b=0$, then the self-energy of such a particle is infinite. If the self-energy of an isolated electron with zero velocity is identified with its rest mass energy $mc^2$ (Marmier and Sheldon, 1969), then $b=1.11\times 10^{-13}$ cm. Generalizing, one might expect the interaction length $b$ to be inversely proportional to total energy. For example, for two charged particles in a real potential there are contributions from each of the two self-energy terms plus a contribution from the interaction potential energy term (Jackson, 1962). Generalizing to a complex potential, one obtains for the proton–electron system of hydrogen

$$b_H=\frac{\pi e^2}{8\left(m_e c^2+m_p c^2+W_{\text{int}}\right)} \tag{33}$$

in which $W_{\text{int}}$ represents the binding energy (energy eigenvalue) of the system. It can be shown that if one solves the time-independent Schrödinger equation for the hydrogen atom using a complex radial coordinate no change in energy eigenvalues will occur. The radial solution is then given by the confluent hypergeometric function $F_{nl}(y)=F(-\lambda_n+l+1,2l+2,y)$ (Landau and Lifshitz, 1965; Merzbacher, 1970).

## 11. MATTER–ANTIMATTER ASYMMETRY

The complex electric potential for a single charged particle is given by

$$\frac{e}{x}=\frac{e}{r^2+b^2}(r-ib)=P_R-iP_I \qquad (34)$$

Postulating the convention that negative $P_I$ describes a source and positive $P_I$ describes a sink (similar to the potential energy case (Schiff, 1968)), let $b$ be positive for electrons and positrons. The electron, with $-|e|$, would have the attributes of a source, and the positron, with $+|e|$, would have the attributes of a sink.

Using the formulas for the number of particles in and the energy of positron and electron gases with $k_B T \gg m_e c^2$ (Landau and Lifshitz, 1969), the energy of a single such particle is $3.14 k_B T$. The interaction length for a single particle is then given by $b=2.1\times 10^{-4}/T$. Evaluating equation (34) at $r=0$, one finds

$$P_I(r=0)=\frac{e}{b}=\mp 2.3\times 10^{-6}T \qquad (35)$$

The core of the positron becomes an increasingly effective sink (plus sign) as $T$ increases. The core of the electron, acting as source (minus sign), maintains the electron in space-time existence as $T$ increases. The number of positrons would become less than the number of electrons near $t=0$, thus introducing an asymmetry between matter and antimatter. If one is confined to the electric force, then $b$ must be chosen negative for protons and antiprotons in order to produce the appropriate asymmetry between them. For the case of neutrons-antineutrons, one could consider that they decay into protons-antiprotons and so contribute slightly (due to their long half-life) to the excess of protons over antiprotons. (Massy particles other than electrons, protons, and neutrons could also lose their antiparticles in the asymmetry mechanism, but they are unstable themselves so they would not remain in excess.)

## 12. $D(t=0)$ FOR VARIABLE $N$

Assume the generalization $d\sigma/c\,dt=[1/N(t)]^{1/4}$ with $N(t)$ no longer a constant within a given era as it was previously and with $N(t)$ decreasing after reaching a maximum of $N$ at $t_1$. The decrease in $N(t)$ would be caused by the asymmetry mechanism of Section 11. Use $k=0$.

(i) Let $N(t)=N/2(1+t/t_1)$, $t_1 \leqslant t \leqslant 0$; then

$$D=\left(\frac{81A_1^2N}{2At_1^4}\right)\frac{1}{(1+t/t_1)^3} \tag{36}$$

$$D(t=0)=\frac{81A_1^2N}{2At_1^4} \tag{37}$$

(ii) Let $N(t)=Ne^{-(1-t/t_1)}$, $t_1 \leqslant t \leqslant 0$; then

$$D=\frac{A_1^2N}{At_1^4}e^{-(1-t/t_1)} \tag{38}$$

$$D(t=0)=\frac{A_1^2N}{eAt_1^4} \tag{39}$$

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